Generalized Bin Packing Problems

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- Packing problems are a class of optimization problems with:
  - a set of “objects”, named items, to be loaded
  - a set of containers or bins which will host the items
  - an objective function to optimize (for instance, minimize the number of bins used)
  - a set of constraints to satisfy (for instance, capacity constraints).
- The main families of packing problems are the Bin Packing and the Knapsack problems
  - in Bin Packing problems all items must be loaded (i.e., all items are compulsory)
  - in Knapsack problems not necessarily must items be loaded (i.e., all items are non-compulsory)

Introduction: packing problems

Bin Packing problems

The most important Bin Packing problems are:

- The Bin Packing Problem (BPP)
  - a set of items with given volumes must be loaded into bins with the same capacity
  - the goal is to minimize the number of bins used
- The Variable Sized Bin Packing Problem (VSBPP)
  - a set of items with given volumes must be loaded into bins with different capacities
  - the goal is to minimize the wasted space
- The Variable Cost and Size Bin Packing Problem (VCSBPP)
  - a set of items with given volumes must be loaded into bins with different capacities and costs
  - the goal is to minimize the overall cost

Introduction: packing problems
Knapsack problems

The most important Knapsack problems are:

- The Knapsack Problem (KP)
  - a set of items with given volumes and profits must be loaded into one bin (called the knapsack)
  - the goal is to maximize the overall profit
- The Multiple Knapsack Problem with Identical capacities (MKPI)
  - a set of items with given volumes and profits must be loaded into a set of knapsacks with the same capacity
  - the goal is to maximize the overall profit
- The Multiple Knapsack Problem (MKP)
  - a set of items with given volumes and profits must be loaded into a set of knapsacks with different capacities
  - the goal is to maximize the overall profit

Why packing problems are so relevant

Packing problems are exploited in many fields and address a lot of applications

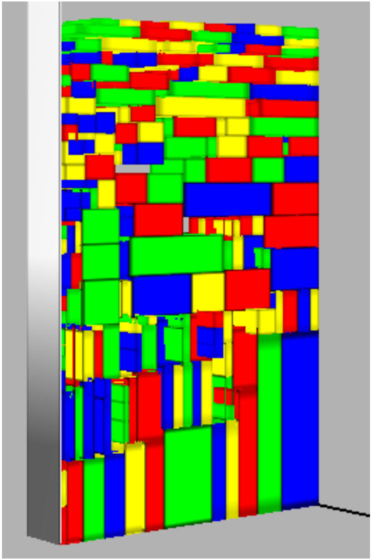
- Computer Science: place computer files (i.e., the items) into memory blocks of fixed size (i.e, the bins)
- Industrial applications: cut several rolls of a material (i.e., the bins) into smaller pieces (customer demands, i.e., the items) in order to minimize the wasted left-overs
- Technology: some Bin Packing algorithms are used in the technology mapping process
- Telecommunications: network traffic is represented by the items. The available channels are represented by the bins.

Why packing problems are so relevant

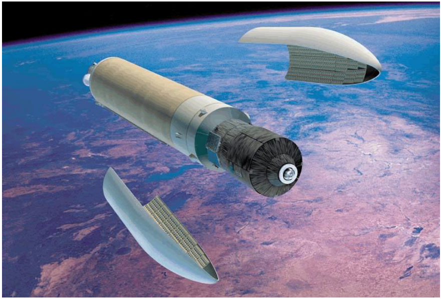
- Economy: in Knapsack problems capacity constraints often play the role of budget constraints
- Transportation and Logistics: freight forwarders, carriers and shipping companies have to arrange shipments in an efficient way
- Aircraft and space cargo loading: items are loaded into freight aircraft or satellites solving specialized knapsack problems
- Last but not least: packing problems are exploited as sub-problems to solve other combinatorial optimization problems.

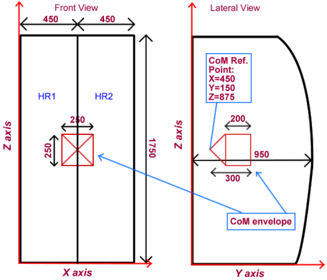
Examples

Examples



Examples




Front View
450
450
HR1
HR2
250
250
1750
Z axis
X axis
Lateral View
CoM Ref. Point: X=450 Y=150 Z=875
200
950
300
CoM envelope
Z axis
Y axis

A new family of packing problems

Although packing problems address a lot of applications, there is a gap in the literature because:

- Nobody has ever considered the joint presence of compulsory and non-compulsory items
- Little has been done in terms of unified methodologies
- Different techniques have been used in order to address different packing problems
- These techniques deal either with compulsory or non-compulsory items
- New real-life applications could be modeled through both compulsory and non-compulsory items

A new family of packing problems

- In order to fill a noteworthy portion of this gap, we formulated a new family of packing problems, named Generalized Bin Packing Problems
- Problems belonging to this new family are characterized by both compulsory and non-compulsory items and multiple item and bin attributes
- These problems are:
  - The Generalized Bin Packing Problem (GBPP)
  - The Stochastic Generalized Bin Packing Problem (S-GBPP)
  - On-line Generalized Bin Packing Problems:
    - The On-line Generalized Bin Packing Problem (OGBPP)
    - The On-line Generalized Bin Packing Problem with item profits proportional to item volumes (OGBPP_κ)
    - The On-line Variable Cost and Size Bin Packing Problem ($\mathrm{OVCSBPP}$)

Our main contributions

Introducing this new family of packing problems we could:

- Fill a relevant part of the gap in the literature regarding packing problems
- Collect very different packing problems into a unique one
- Use unified and efficient techniques to address all these problems
- Model new applications not yet addressed by the previous packing problems

In the GBPP:

- Items are characterized by multiple attributes: volume and profit
- Items can be compulsory and non-compulsory
- Bins are characterized by multiple attributes: capacity and cost
- The goal is to minimize the overall cost i.e., bin costs minus item profits
- Capacity constraints must be satisfied
- Possibility to express bin usage constraints (they arise in freight forwarders settings)

The GBPP addresses a noteworthy number of applications:

- In Transportation and Logistics: the GBPP is able to describe the trade-off between shipping costs and item profits
- In Airfreight Transportation items are loaded according to their volume
- In the Waste Collection Problem: the GBPP is a design problem where vehicles are determined at a tactical level

The GBPP yields a relevant number of contributions:

- it is the first packing problem dealing with both compulsory and non-compulsory items and with multiple attributes
- we can gather several packing problems into a unique one. These problems are:
  - The BPP
  - The VSBPP
  - The VCSBPP
  - The KP
  - The MKPI
  - The MKP

The Generalized Bin Packing Problem
Methodological contributions

- We gave two formulations of the GBPP
- We proposed ad hoc methodologies to address the problem in terms of accuracy and efficiency
- Therefore, we can use the same techniques to address all problems described by the GBPP.
- This avoids the need to change technique every time the setting changes
- We created new instance sets to test these techniques
- The techniques we studied were:
  - accurate lower bounds with an overall gap of 0.08%
  - accurate upper bounds with an overall gap of 0.11%
  - an exact method called branch-and-price with an overall gap of 0.03%
  - an approximate method called beam search with an overall gap of 0.75%
- We could solve most instances, for the first time.

In order to test the proposed techniques we produced four classes of instances:

- Class 0: 300 instances; the same as Monaci (2002) for tackling the VSBPP; compulsory items only
- Class 1: 300 instances; non-compulsory items with profits drawn from a uniform distribution as $p_i \in \lceil \mathcal{U}(0.5, 3) w_i \rceil$
- Class 2: 300 instances; non-compulsory items with profits drawn from a uniform distribution as $p_i \in \lceil \mathcal{U}(0.5, 4) w_i \rceil$
- Class 3: 60 instances; both compulsory and non-compulsory items

The Generalized Bin Packing Problem

Computational Results: lower bounds

	LB_1		LB_2		LB_3	
CLASS	% GAP	OPT	% GAP	OPT	% GAP	OPT
0	0.53	160 (53%)	0.11	74 (25%)	0.06	216 (72%)
1	0.78	27 (9%)	0.12	81 (27%)	0.10	99 (33%)
2	0.52	36 (12%)	0.09	72 (24%)	0.07	99 (33%)
OVERALL	**0.61**	**223 (25%)**	**0.11**	**227 (25%)**	**0.08**	**414 (46%)**

The Generalized Bin Packing Problem

Computational Results: constructive heuristics

CLASS	FFD(1) % GAP	FFD(2) % GAP	FFD(3) % GAP	FFD(4) % GAP
0	11.15	11.15	1.66	1.66
1	10.93	15.50	2.32	7.26
2	8.03	9.80	1.86	3.66
OVERALL	**10.04**	**12.15**	**1.95**	**4.19**

CLASS	BFD(1) % GAP	BFD(2) % GAP	BFD(3) % GAP	BFD(4) % GAP
0	11.15	11.15	1.62	1.62
1	10.88	15.38	2.30	7.22
2	8.00	9.77	1.81	3.63
OVERALL	**10.01**	**12.10**	**1.91**	**4.16**

The Generalized Bin Packing Problem

Computational Results: upper bounds

CLASS	BFD(3) % GAP	L-BFD(1, 0.1, 3) % GAP	C-BFD(3) % GAP	z_{SC} % GAP
0	1.62	1.93	1.27	0.14
1	2.30	2.52	1.93	0.10
2	1.81	2.08	1.56	0.07
OVERALL	**1.91**	**2.18**	**1.58**	**0.11**

CLASS	DIVE(1) % GAP	DIVE(2) % GAP	B-DIVE(1) % GAP	B-DIVE(2) % GAP
0	1.30	0.98	0.65	0.62
1	0.95	1.37	0.51	0.55
2	0.85	1.32	0.42	0.49
OVERALL	**1.04**	**1.22**	**0.53**	**0.55**

The Generalized Bin Packing Problem

Computational Results: class 3 upper bounds

% COMPULSORY ITEMS	BEST BFD % GAP	Z_{SC} % GAP	BEST DIVING % GAP
0	0.77	0.12	0.37
25	1.73	0.51	1.00
50	12.21	2.72	7.52
75	2.04	0.52	1.15
100	0.85	0.16	0.51
MEAN	**3.52**	**0.81**	**2.11**

% COMPULSORY ITEMS	BEST BFD (seconds)	Z_{SC} (seconds)	BEST DIVING (seconds)
0	< 0.01	11.37	0.39
25	< 0.01	11.94	0.55
50	< 0.01	13.95	0.42
75	< 0.01	13.41	0.33
100	< 0.01	13.42	0.25
MEAN	$<$ **0.01**	**12.82**	**0.39**

The Generalized Bin Packing Problem

Computational Results: branch-and-price for Classes 0, 1, and 2

CLASS	% GAP(0)	% GAP	NODES	OPT	ROOT OPT	TIME
0	0.17	0.04	1101.03	247	150	603.97
1	0.13	0.03	1593.27	226	123	859.60
2	0.09	0.03	1570.32	229	136	859.51
OVERALL	**0.13**	**0.03**	**1421.54**	**702**	**409**	**774.36**

The Generalized Bin Packing Problem

Computational Results: branch-and-price for Class 3 instances

% COMPULSORY ITEMS	% GAP(0)	% GAP	NODES	OPT	ROOT OPT	TIME
0	0.11	0.10	1291.33	3	1	2820.44
25	0.32	0.31	1109.00	4	3	2472.01
50	2.11	1.86	1058.50	4	1	2525.91
75	0.47	0.41	1080.17	4	0	2749.93
100	0.21	0.15	1234.33	4	1	2626.93
OVERALL	**0.65**	**0.57**	**1154.67**	**19**	**6**	**2639.04**

The Generalized Bin Packing Problem

Computational Results: beam search results

CLASS	BEAM	% GAP	OPT	IMPROVING	TIME
0	1	0.33	130	2	23.35
	2	0.29	150	3	28.59
	3	0.28	159	3	31.12
	4	0.26	170	3	33.94
		0.29	**176**	**4**	**29.25**
1	1	1.25	99	3	39.29
	2	1.16	109	3	54.10
	3	1.10	114	2	59.71
	4	0.98	124	2	64.58
		1.12	**128**	**3**	**54.42**
2	1	0.93	103	4	42.43
	2	0.84	113	3	53.64
	3	0.79	119	2	60.22
	4	0.74	123	2	65.89
		0.83	**129**	**4**	**55.54**
3	1	4.97	7	1	145.74
	2	4.72	9	0	155.54
	3	4.70	11	1	157.95
	4	4.68	11	1	158.63
		4.77	**11**	**2**	**154.47**
OVERALL		**1.75**	**444**	**13**	**73.42**

ITEMS	BB_{HS} % GAP	BB_{HS} OPT	B&P % GAP	B&P OPT
25	0	60	0	60
50	0.01	59	0	60
100	0.02	59	0.1	57
200	0	60	0.6	41
500	0	60	0.11	29

ITEMS	VNS_{HSB} % GAP	VNS_{HSB} OPT	VNS_{HSB} TIME	BEAM % GAP	BEAM OPT	BEAM TIME
25	0.00	60	150	0.09	54	0.10
50	0.01	59	150	0.21	45	0.53
100	0.00	58	150	0.32	35	3.60
200	0.01	54	150	0.28	20	37.03
500	0.01	52	150	0.41	22	128.44

In the S-GBPP:

- Items are characterized by multiple attributes: volume and profit
- Bins are characterized by multiple attributes: cost and capacity
- Each item profit depends on the bin where the item is loaded
- Each item profit is described by a random variable with unknown probability distribution
- The goal is to maximize the expected total net profit: expected total profit minus total bin cost
- Capacity constraints must be satisfied
- Possibility to express bin usage constraints (they arise in freight forwarders settings)

The Stochastic Generalized Bin Packing Problem

The S-GBPP addresses new applications

- In Transportation and Logistics
  - Freight consolidation takes place during the delivery process
  - Profits are random terms which represent a series of handling operations performed at logistic platforms
  - These operations could significantly affect the final total profit of the loading
- In Railway Maintenance Management
  - This problem is currently being exploited in the AcemRail project
  - Maintenance operations (named warnings, i.e., the items) must be scheduled in timeslots (i.e., the bins)
  - Warning costs are uncertain and depend on the timeslot to which they are assigned

The S-GBPP yields a relevant number of contributions:

- It generalizes a richer setting
- Often, probability distributions are not known a priori
- Methodological contributions:
  - We provided a stochastic model
  - We could derive an approximated deterministic model applying the extreme value theory

- The OGBPP works like the GBPP but the items are not known a priori
- They arrive on-line to a decision maker
- Only when an item arrives is its information revealed
- In most applications, item profits are proportional to their volumes. Therefore, we also studied the On-line Generalized Bin Packing Problem with item profits proportional to item volumes (OGBPP_κ)
- Finally, as nobody has ever tackled the on-line variant of the VCSBPP, we also studied the On-line Variable Cost and Size Bin Packing Problem ($\mathrm{OVCSBPP}$)

On-line Generalized Bin Packing Problems address a relevant number of applications:

- Transportation and Logistics: usually, shipping orders are not known a priori but arrive online to freight forwarders
- In this setting, non-shipped items will be deferred to the next shipment
- All those settings where items arrive online

- This is the first time the on-line variant of the VCSBPP and GBPP has been studied
- We proposed two algorithms (named First Fit (FF) and Best Fit (BF)) for the OVCSBPP and we proved that a worst case ratio bound of value 2 exists
- We showed that no worst-case ratio bound can be computed for FF and BF when applied to the OGBPP
- We studied improved variants of the FF and BF and we showed that the same conclusion holds, even for the OGBPP_{κ}
- We proved a very strong result: no worst-case ratio bound can be computed even for two popular *off-line* algorithms, First Fit Decreasing (FFD) and Best Fit Decreasing (BFD) when applied to the OGBPP and to the OGBPP_{κ}

Conclusions and future research

Conclusion

- We introduced a new family of packing problems: the Generalized Bin Packing Problems
- These problems are characterized by:
  - the joint presence of compulsory and non-compulsory items
  - multiple attributes
- Our main results:
  - addressing new applications and providing new contributions
  - collecting very different packing problems into a unique one
  - using the same techniques to address all these problems
- The proposed techniques are very flexible, effective, and efficient

Conclusions and future research

Future work

- Propose improved algorithms for the OVCSBPP
- The On-line Generalized Bin Packing Problem is still an open problem in terms of worst-case analysis:
  - we proved that no worst-case ratios can be computed for some well-known on-line and off-line algorithms
  - research question: does an on-line algorithm for the OGBPP with a worst-case ratio exist?

Publications

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